A Perturbational Technique for the Fast Modelling of Printed Reflectarray Elements

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Abstract: This paper presents a novel method for the element characterization of printed reflectarray antennas. Their characterization requires the calculation of the phase of the reflection coefficient versus the patch dimension (phase diagram), when the structure is illuminated by a uniform plane wave. The calculation of the phase diagram is based on the full–wave analysis of the structure in a limited number of points, and a polynomial interpolation of the phase, which takes advantage of a perturbational technique for estimating the phase derivative. This approach significantly reduces the computing time for the calculation of the phase diagram, as shown through an example.

Keywords: Frequency Selective Surfaces, Periodic structures, Perturbation Technique, Printed Reflectarray Antennas.

1 Introduction

Printed reflectarrays antennas received a lot of attention in the last decade as a valid alternative to classical reflector antennas, for both scientific and commercial applications [1]–[5]. Similarly to reflector antennas, reflectarrays comprise a low–gain feed, located in front of a reflecting surface. The peculiarity of reflectarrays is that the reflector is a planar array of printed patches on a grounded dielectric substrate (Fig. 1). In order to obtain the focussing effect, the patches have slightly different size and/or shape, thus leading to a proper variation of the phase of the local reflection coefficient [1].

Different techniques have been proposed for phase shifting the local reflection coefficient: among them, the most popular are based on the use of identical microstrip patches with different length phase–delay lines [1, 3], variable size microstrip patches [2, 5], and identical patches

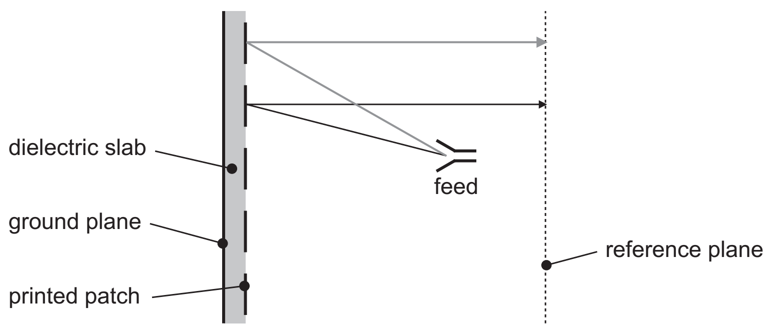

Figure 1: Principle of operation of a printed reflectarray.

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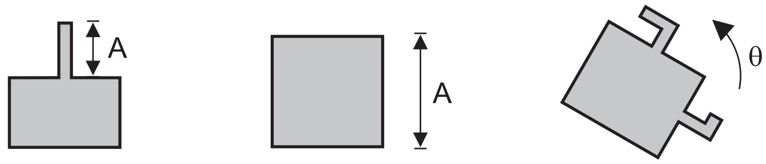


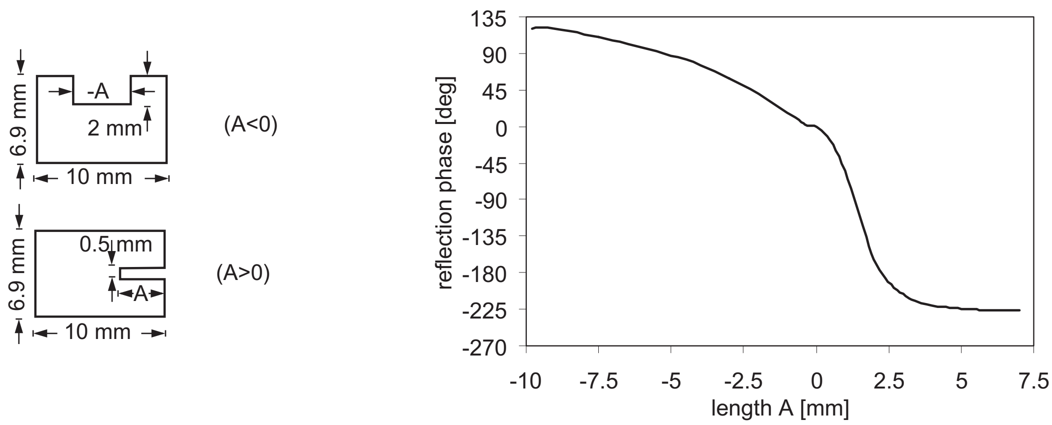
Figure 2: Classical shapes used in printed reflectarrays.

with different rotation angles [4] (Fig. 2). The choice of the element shape is a key issue in the design of reflectarray antennas. The performance of different element shapes have been compared in a systematic way in [6].

The major advantages of printed reflectarray antennas are low profile, reduced weight, and easy manufacturing. Moreover, especially in spaceborne systems, printed reflectarray can be folded for launching and deployed for operation. The only significant disadvantage is the narrow bandwidth of these antennas, typically limited to few percent.

In the design of printed reflectarrays, geometrical optics is usually applied for the calculation of the electrical length of the ray path from the feed to the reflector and from the reflector to the reference plane (Fig. 1). Conversely, a full–wave analysis is applied for the calculation of the phase shift due to the reflection. This analysis is typically performed under the local periodicity approximation: each metal patch is considered in a periodic array of identical elements, thus neglecting the variation of mutual coupling due to the small difference between neighboring elements. The output of the full–wave analysis is the “phase diagram”, which is the plot of the phase of the reflection coefficient versus the geometrical dimension A of the array element (Fig. 3). Once the path lengths and the phase diagram have been calculated, the proper element dimension is selected, in order to create the required radiation pattern of the reflectarray antenna.

Whichever is the adopted full–wave approach, the calculation of the phase diagram is the most CPU intensive part of the analysis. In this paper, we propose the use of an interpolation technique, which is based on a perturbational approach in conjunction with an integral method, named the MoM/BI–RME method. This technique allows for a substantial reduction in CPU time.



a *b*

Figure 3: Example of a phase diagram: *a*) geometry of the patch and geometrical parameter A; *b*) phase of the reflection coefficient versus A , in the case of normal incidence, vertical polarization.

2 Phase diagram calculation by the MoM/BI–RME method

In our approach, the determination of the phase diagram is based on the MoM/BI–RME method. This method consists in the solution of an electric–field integral equation by the Method of Moments (MoM) with entire–domain basis functions. The vector basis functions are related to scalar functions ψ_i, which are the first N eigenfunctions of the Helmholtz equation in a domain S coincident with the cross–section of the patch

$$\nabla^2\psi + \kappa^2\,\psi = 0 \qquad \text{in } S \tag{1}$$

with Dirichlet ($\psi = 0$) or Neumann ($\partial\psi/\partial n = 0$) boundary condition on ∂S ($\vec{n}$ is the outward normal unit vector on ∂S). The eigenfunctions ψ_i and the corresponding eigenvalues κ_i are numerically calculated by the Boundary Integral–Resonant Mode Expansion (BI–RME) method, which applies to printed elements with an arbitrary shape [7].

The MoM/BI–RME method was originally developed for the analysis of frequency selective surfaces (FSS) [8, 9]. In that case, the method provides the frequency response of the FSS for a given geometry. Its application requires two different steps: the determination of the basis functions, which is frequency independent, and the solution of the MoM problem, which is performed frequency–by–frequency (Fig. 4*a*). The major advantage of this method derives from the use of entire–domain basis functions, which must be calculated only once and lead to a MoM matrix problem with a small number (N) of unknowns.

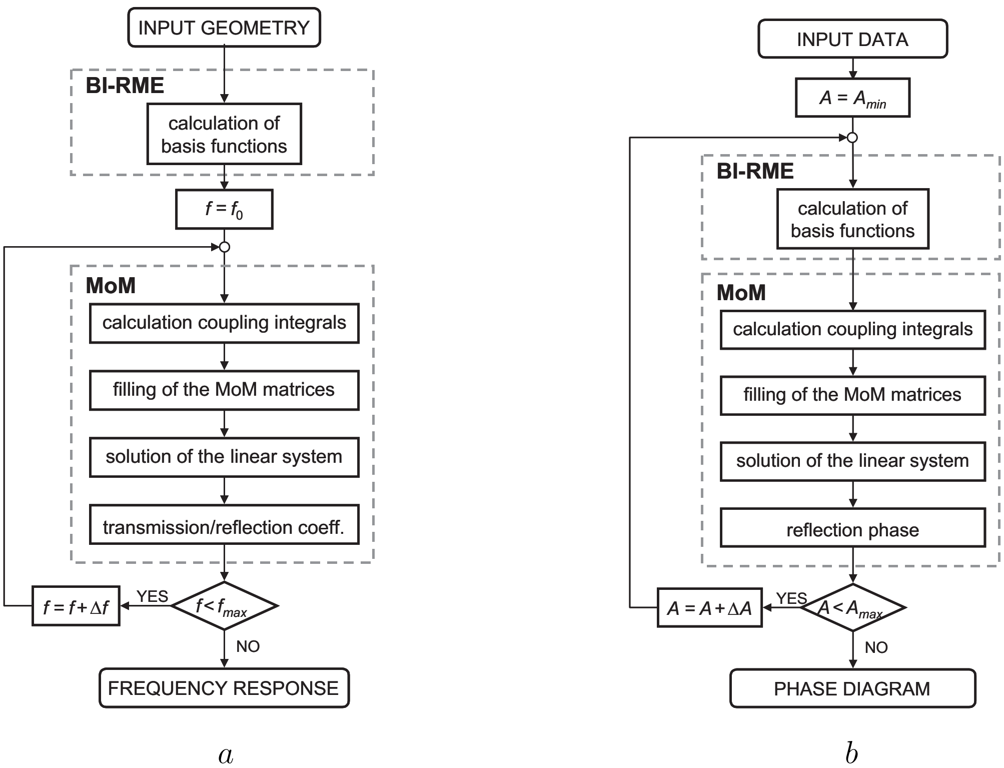


Figure 4: Flow–chart of the MoM/BI–RME method: *a*) calculation of the frequency response of frequency selective surfaces; *b*) calculation of the phase diagram of printed reflectarrays.

The typical CPU time for the calculation of the frequency response of FSS in 100 points is in the order of $5 \div 6$ seconds. This CPU time is almost equally balanced between the calculation of the basis functions and the frequency loop [8].

The analysis of reflectarrays requires a slightly different approach. In fact, the calculation of the phase diagram is performed at a single frequency, by varying the geometry of the patch. This implies that the entire–domain basis functions must be calculated for each value of the geometrical parameter A (Fig. 4b), thus weakening the advantage of using the MoM/BI-RME method.

An example of phase diagram calculated by the MoM/BI–RME method is reported in Fig. 3, in the case of a patch element with ridged rectangular shape.

3 Interpolation method and perturbational technique

A significant reduction in computing time can achieved by using an interpolation technique. Since the phase diagram is typically a smooth function of the geometrical dimension A (see Fig. 3), it could be well fitted by using interpolating polynomial functions. In particular, the range of variation of A is divided into a number of identical intervals. In each interval $[\underline{A}, \overline{A}\,]$, the phase diagram is represented as a third order polynomial function

$$\phi(A) = c_0 + c_1\, A + c_2\, A^2 + c_3\, A^3 \qquad\qquad \text{in } [\underline{A}, \overline{A}\,] \tag{2}$$

and the coefficients $c_0,\ \ldots,\ c_3$ are determined from the knowledge of the phase ϕ and its derivative $d\phi/dA$ in $\underline{A}$ and $\overline{A}$. It is found that few intervals (usually less than ten) are needed for an accurate interpolation (error smaller than 1 deg). The use of this technique significantly reduces the number of analyses, but requires an additional effort for the calculation of the phase derivative. The derivative could be obtained numerically by the calculation of the phase in two close points, namely A and $A' = A(1+\varepsilon)$, with $|\varepsilon| \ll 1$. Nevertheless, even in this case the use of interpolating functions is advantageous over the standard approach.

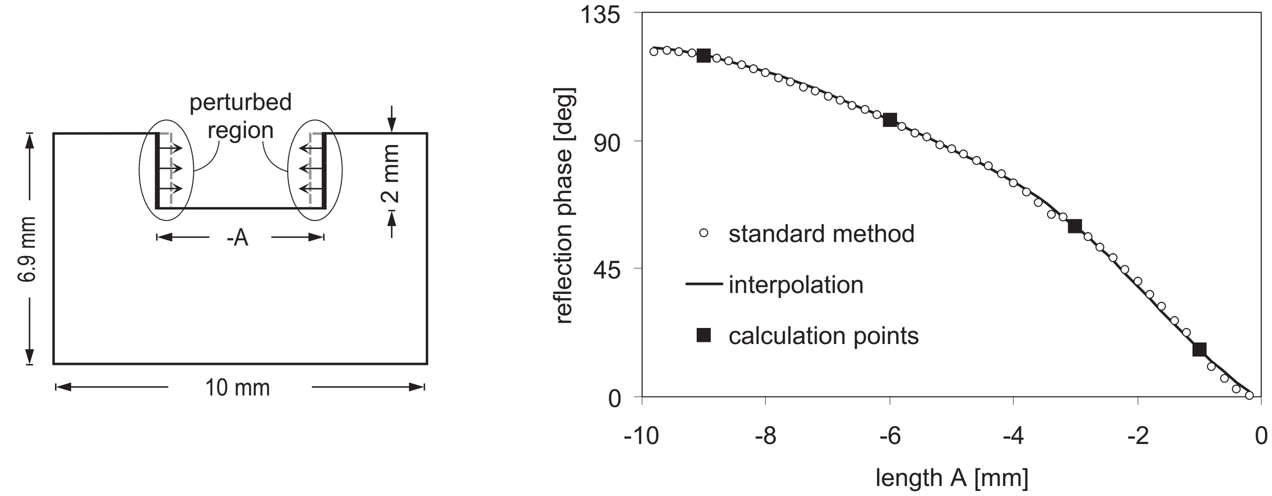
Another significant improvement in computational efficiency can be accomplished by adopting a perturbational technique in conjunction with the MoM/BI–RME method. In fact, being ψ_i the functions on a patch with cross–section S (corresponding to a given value of A), functions ψ'_j in S' (corresponding to A') can be obtained as a linear combination of the N functions ψ_i

$$\psi'_j = \sum_{i=1}^{N} x_{ij}\ \psi_i. \tag{3}$$

The unknown coefficients x_{ij} are obtained as follows. A variational formula for κ' (which represent the eigenvalues of the Helmholtz equation in the domain S') is used [10]

$$\kappa'^2 = \frac{\displaystyle\int_{S'} \nabla\psi' \cdot \nabla\psi'\ dS - 2\beta \int_{\partial S'} \psi'\ \frac{\partial \psi'}{\partial n'}\ d\ell}{\displaystyle\int_{S'} \psi'^2\ dS} \tag{4}$$

where $\beta = 1$ or $\beta = 0$ in the case of Dirichlet or Neumann boundary condition, respectively. By substituting (3) into (4), and applying Rayleigh-Ritz method to find the stationary value of the resulting expression, a generalized linear eigenvalue problem with dimension N is obtained. The



a *b*

Figure 5: Analysis of the ridge-shape element by the perturbational technique: *a*) geometry of the element and perturbed region; *b*) phase diagram of the ridge element, calculated by the standard method (dots) and by the interpolation procedure (solid line).

solution of this problem yields the set of κ'_j as eigenvalues, and coefficients x_{ij} as the corresponding eigenvectors. Since the dimension N of this eigenvalue problem is small, the new basis functions ψ'_j can be obtained with a limited computational effort, much smaller than the standard calculation by the BI–RME method.

The perturbational approach can be also exploited in the MoM portion of the code (Fig. 4*b*). In fact, the coupling integrals in the modified domain S' (between the basis functions and the Floquet modes) can be written as a linear combination of the coupling integrals in the original domain S, plus line integrals on the modified portion of the boundary. In such a way, the most CPU–intensive part of the application of the MoM can be completely skipped, and only the solution of a (small) linear system must be performed.

4 Numerical results

As an example, the phase diagram of the ridge element, already determined by the standard technique (Fig. 3*b*), is calculated by using the MoM/BI–RME method in conjunction with the perturbational approach. Fig. 5*a* shows the structure and the portion of the boundary which is displaced for the calculation of the phase derivative. The phase diagram is reported in Fig. 5*b*. The analysis was performed in only four points (squared markers), where the reflection phase is calculated by the MoM/BI–RME method and the phase derivative is obtained by the perturbational technique. The interpolated phase diagram (solid line) practically coincides with the one calculated in 49 points by the standard method (dots). The CPU time was 18 sec by the new approach, instead of 142 sec by the standard method (on a PC Pentium IV @ 1.4 GHz).

5 Conclusion

This paper presented a novel technique for the fast modelling of printed reflectarray elements with arbitrary shapes. The proposed approach combines the MoM/BI–RME method (used for the full–wave analysis of the structure) with a perturbational technique, which allows for obtaining an interpolated plot of the phase diagram in a significantly reduced computing time. An example demonstrated the effectiveness of the proposed method.

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